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Compressive buckling behavior of laminates with interleaves of very low elastic modulus

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Compressive buckling behavior of laminates with interleaves of very low elastic modulus

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Abstract—The damping capability of fiber reinforced composite laminates can be remarkably improved by interleaving damping material sheets. However, the compressive buckling property reduces significantly due to the existence of interleaves of very low elastic modulus. The buckling loads of the composite plates with interleaves are explicitly given by assuming that only the interleaves deform due to transverse shear. The results are compared with finite element results and are found to agree well with those from a finite element analysis in spite of its simplicity. The buckling load reduction can be estimated through a nondimensional shear rigidity parameter of the interleaves.

Keywords: Laminate; interleave; compressive buckling; transverse shear.

1. INTRODUCTION

Damping properties of flexible space composite structures are required to be quite high due to the absence of air damping and stringent requirements for high quality in space missions. A material system made of composite laminates interleaved with more than one damping material sheet has been proposed by Fujimoto *et al.* [1, 2]. The damping properties of the composite laminates are remarkably improved by the introduction of many interleaves [1, 3]. Mechanical properties of composite laminates governed by low interlaminar toughness such as impact resistance, fatigue, etc. are also improved by interleaving soft damping material sheets, because the interleaves reduce the interlaminar stresses at the edge of laminates and stress concentration at delamination fronts. However, the compressive buckling loads of the interleaved laminates are found to significantly reduced due to the introduction of such material sheets of very low modulus, whose order is 10^{-3}–10^{-4} of the modulus of the base layers. Since this reduction of compressive buckling load may be fatal to structural members, the mechanism and the amount of the reduction of buckling properties due to interleaving such a material need to be disclosed.

The buckling property characteristic of laminates with more than one interleave is studied analytically and numerically and a simple closed-form equation for roughly estimating the buckling property reduction is derived in order to give a design guideline for interleaving soft material sheets between laminae.

2. THEORY

In this section, an analytical development is performed to obtain a simple method to estimate buckling loads of laminated plates with very soft interleaves. The laminate is assumed to be infinitely long in transverse direction. The analytical model is shown in Fig. 1. The laminate fixed at both loading ends is made of $2N$ stiff base layers and $2N - 1$ very flexible interleaves and has repeating arrangement with interleaves

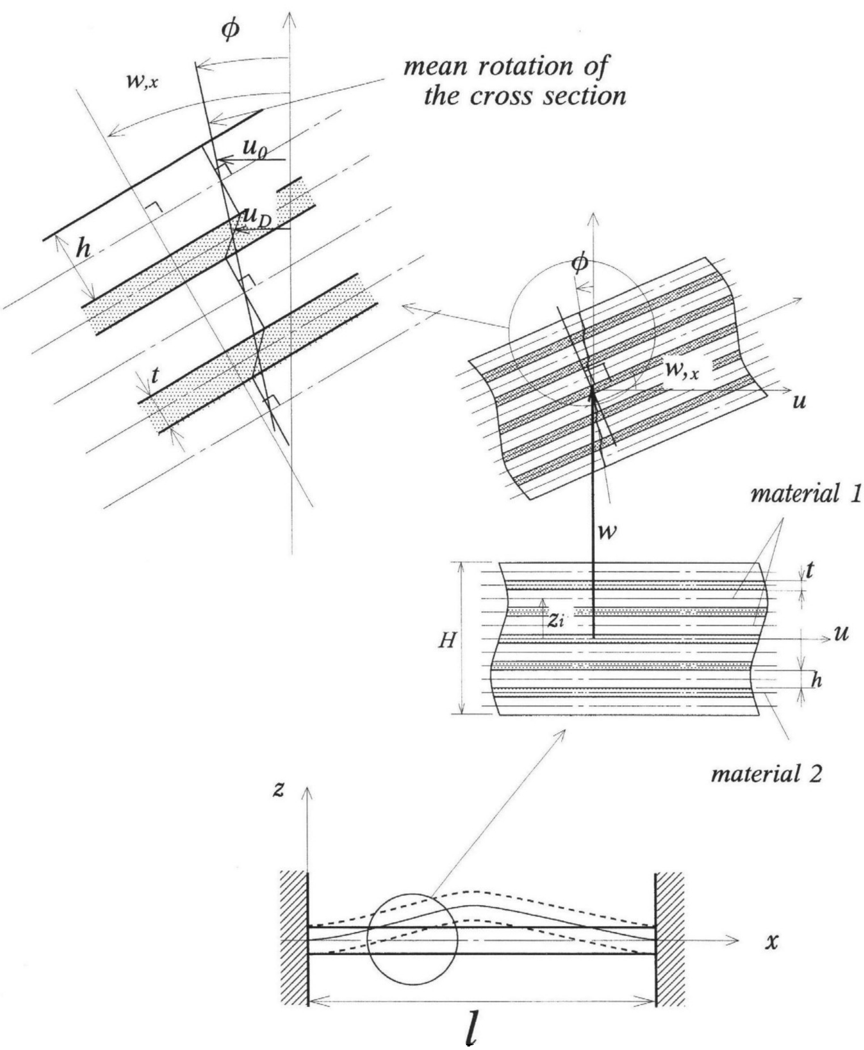

Figure 1. Analytical model and the assumption of the displacement field.

located between two composite layers. The base layers and the interleaves have common thickness h and t, respectively.

Owing to the discontinuity of the material properties, the displacement distribution in the cross-section is quite complex. However, since the shear deformation of the interleaves must be very large compared with that of the base layers, the shear deformation of the base layers is neglected here. When the modulus of the interleaves approaches zero, the constraint of the interleaves becomes weak and each base layer deforms like an isolated bar. Then, the mean rotation of the cross section approaches zero, as shown in Fig. 2. So, in this extreme case, as the longitudinal displacements u_0 of all the middle planes of the stiff layers in a cross section becomes zero, the transverse shear deformations of the interleaves γ_{xz} are also same in a cross section and

$$\gamma_{xz} = \frac{h}{t}\frac{\mathrm{d}w}{\mathrm{d}x},$$

where w is a transverse deflection and h and t are the thicknesses of the stiff base layer and soft interleaves. However, when the stiffness of the soft interleaves is not small, the effect of the shear deformation on buckling behaviors becomes negligible. In this

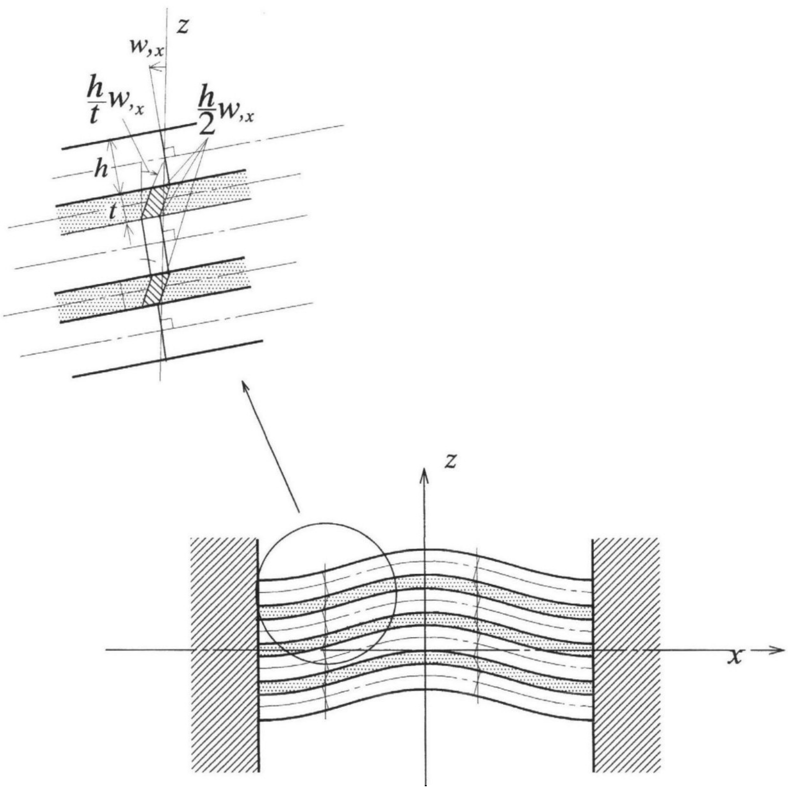

Figure 2. The deformed shape of the cross section when the stiffness of the interleaves is infinitely small. The base layers deflect by the same amount and all the interleaves have the same shear deformation in a cross section.

first attempt, the shear strain in all the interleaved soft material sheets is assumed to be uniform in the thickness direction for convenience. This approximation may not be very accurate when the shear deformation changes in the thickness direction, but this assumption makes the problem simple enough to get a closed-form solution. The validity of the results will then be checked by a finite element analysis.

The displacement fields $u_{0i}(x, z)$ and $u_{Dj}(x, z)$ of both the ith base layers ($i = -N, -N+1, \ldots, -1, 1, \ldots, N-1, N$) and the jth flexible interleaves ($j = -N+1, -N+2, \ldots, 0, \ldots, N-1$) in the longitudinal direction can be written with the deflection $w(x)$ of middle plane of the plate and the mean rotation of the cross section $\phi(x)$ as

$$u_{0i} = -(z - z_i)w' - z_i\phi, \tag{1}$$

$$u_{Dj} = \frac{h}{t}\{z - j(h+t)\}w' - \frac{h+t}{t}(z - jh)\phi, \tag{2}$$

where

$$z_i = \frac{2i-1}{2}(h+t).$$

The variables x and z are coordinates in the longitudinal direction measured from a fixed end and in the thickness direction measured from the middle plane of the laminates, respectively and (′) denotes a differentiation with respect to x. The suffixes 0 and D refer to the stiff base layer and the very soft interleave, respectively. The deflection w is assumed to be constant in a cross section ($w = w(x)$), that is, w is independent of the z-coordinate.

The strain distributions in the stiff base layers and in the soft interleaves are obtained by substituting equations (1) and (2) into ordinary strain–displacement relations of small displacement problem.

$$\begin{aligned} \varepsilon_{x0i} &= \frac{\partial u_{0i}}{\partial x} = -(z - z_i)w'' - z_i\phi', \\ \gamma_{xz0i} &= \frac{\partial u_{0i}}{\partial z} + \frac{\partial w}{\partial x} = 0, \end{aligned} \tag{3}$$

$$\begin{aligned} \varepsilon_{xDj} &= \frac{\partial u_{Dj}}{\partial x} = \frac{h}{t}\left[\{z - j(h+t)\}w'' - \left(1 + \frac{t}{h}\right)(z - jh)\phi'\right], \\ \gamma_{xzDj} &= \frac{\partial u_{Dj}}{\partial z} + \frac{\partial w}{\partial x} = \frac{h+t}{t}(w' - \phi). \end{aligned} \tag{4}$$

Since w is uniform in the cross section and the longitudinal displacement field u is anti-symmetric with respect to middle plane of the plate, only the upper half of the plate need be considered and total strain energy is

$$\begin{aligned} U = \int_0^l \Bigg\{ & 2\sum_{i=1}^{N} \int_{z_i-h/2}^{z_i+h/2} \frac{1}{2} E_0' \varepsilon_{x0i}^2 \, dz \\ & + \sum_{j=0}^{N-1} k_j \int_{j(h+t)-t/2}^{j(h+t)+t/2} \frac{1}{2}(E_D' \varepsilon_{xDj}^2 + G_D \gamma_{xzDj}^2)\, dz \Bigg\} dx, \end{aligned} \tag{5}$$

where $E_0' = E_0/(1-\nu_0^2)$ and $E_D' = E_D/(1-\nu_D^2)$ and E, ν and G denote Young's modulus, Poisson's ratio and shear modulus, respectively.

$$k_j = \begin{cases} 1 & \text{when} \quad j = 0, \\ 2 & \text{when} \quad j \geqslant 1. \end{cases}$$

The buckling mode should be similar to that for a uniform beam with clamped ends. Therefore, the deformation shape is assumed to be as follows.

$$w(x) = hW\left(1 - \cos\frac{2\pi x}{l}\right), \tag{6}$$

$$\phi(x) = \frac{2\pi h}{l} q \sin\frac{2\pi x}{l}, \tag{7}$$

where W and q are nondimensional generalized coordinates to be obtained. By substituting equations (6) and (7) into equation (5), we can express the strain energy as a quadratic function of the generalized coordinates W and q

$$\begin{aligned} U =& \frac{2\pi^4 N E_0' h^5}{3l^3}\Big[\{1 + e_D + pg_D\}W^2 - 2\{\alpha e_D + pg_D\}qW \\ &+ \{p + (2N-1)^2\alpha^2 e_D + pg_D\}q^2\Big], \end{aligned} \tag{8}$$

where

$$e_D = \frac{(2N-1)E_D' t}{2NE_0' h}, \tag{9}$$

$$g_D = \frac{6G_D l^2}{\pi^2 2N(2N+1)E_0' ht}, \tag{10}$$

$$\alpha = \frac{h+t}{h}, \qquad p = (4N^2-1)\alpha^2.$$

The parameter g_D is a nondimensional effective shear rigidity of the interleaves. The external work V done by the applied compressive load per unit width P is given as

$$V = \frac{1}{2}P\int_0^l (w')^2\,dx = \frac{\pi^2 h^2}{l}PW^2. \tag{11}$$

Using the theory of minimum potential energy ($\Pi = U - V$), we have simultaneous equations with respect to q and W. The eigenvalue problem can be solved explicitly and the buckling load P_{cr} is written as

$$P_{cr} = \frac{2\pi^2 N E_0' h^3}{3l^2}\left[1 + e_D + pg_D - \frac{(\alpha e_D + pg_D)^2}{p + pg_D + (2N-1)^2 e_D}\right]. \tag{12}$$

Assuming that Kirchhoff's hypothesis is valid and no shear deformation occurs, the buckling load P_{cr0} is obtained by using the second moment of area of the laminate.

$$P_{cr0} = \frac{2\pi^2 N E_0' h^3}{3l^2}\left[1 + p + e_D\left\{(2N-1)^2\alpha^2 + 1 - 2\alpha\right\}\right]. \tag{13}$$

The following nondimensional buckling load is introduced to clearly express the reduction of the buckling load from the ideal buckling load P_{cr0}.

$$S_{cr} = \frac{P_{cr}}{P_{cr0}}. \tag{14}$$

When the shear rigidity of the interleaves is high and the shear deformation of the damping sheet is small enough, S_{cr} approaches unity. When $e_D \ll 1$, we have the following expression by making $e_D \to 0$.

$$S_{cr} \approx \frac{(1+p)^{-1} + g_D}{1 + g_D}. \tag{15}$$

When the thickness of damping sheet is thin compared with the stiff base layers, we may use the following approximation.

$$1 + p = 1 + 4N^2\alpha^2 - \alpha^2 \approx 4N^2\alpha^2.$$

Then, we have the following estimation of reduced buckling load of the laminate with soft interleaves.

$$S_{cr} \approx \frac{(2N\alpha)^{-2} + g_D}{1 + g_D} = \frac{1}{4N^2\alpha^2} + \frac{g_D}{1 + g_D}\left(1 - \frac{1}{4N^2\alpha^2}\right). \tag{16}$$

The simplified equation shows that the buckling load of a laminate with very soft interleaves ($e_D \ll 1$) reduces with increase of the number of interleaves N and is governed by the nondimensional effective shear stiffness parameter g_D. When $g_D \ll 1$, the buckling load of the plate approaches a sum of the buckling load of the individual base layers ($S_{cr} \to 1/(4N^2\alpha^2)$). On the other hand, when $g_D \gg 1$, the normalized buckling load approaches unity, that is, the buckling load is equal to the value obtained based on the Kirchhoff's hypothesis.

3. FINITE ELEMENT ANALYSIS

In order to confirm the accuracy of the present theory, a finite element analysis (NISA II) is performed. The element used is an 8-node isoparametric element. A plane strain condition is assumed. A nonlinear analysis is performed by a displacement increment technique. A typical finite element mesh is shown in Fig. 3.

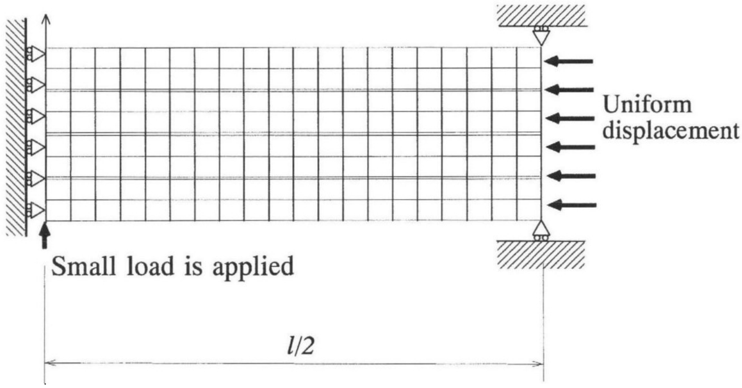

Figure 3. A typical finite element mesh, when $2N = 4$.

Table 1.
Material properties and dimensions of base layers and interleaved sheets (relative stiffness factor of the interleave $c = 10^{-4}$, 10^{-3}, 10^{-2}, 10^{-1} and 1)

Base layer		Interleaved sheets	
E_0	100 GPa	E_D	$c \times E_0$
ν_0	0.3	ν_D	0.4
$2N \times h$	4 mm	t	0.05 mm
l	40 mm, 80 mm, 160 mm, 320 mm, 640 mm		

The buckling load is obtained following the Southwell method from the relation between compressive load and center deflection. The material properties and dimensions of the model are listed in Table 1.

In Fig. 4, the nondimensional load $S = P/P_{cr}$ is plotted against the normalized center deflection w/h when $2N = 4$ and $l = 160$ mm. The buckling load reduction becomes significant when the stiffness of the interleaves is smaller than 10^{-3} of that of the base layer.

4. RESULTS AND DISCUSSIONS

In Figs 5, 6 and 7, the normalized buckling load S_{cr} is plotted against the plate length l for $N = 2, 4$ and 8, respectively. The buckling load reduction from the ideal laminate becomes more significant when the length of the laminate is shorter. The curve shifts to the left with increase of the stiffness of the interleaves. The effect of the transverse shear becomes more important when the length of the plate decreases and/or the stiffness of the interleaves is low. The present results agree well with the finite element results when the stiffness of the interleaves is very low.

The nondimensional buckling load obtained from the finite element analysis, when $E_D/E_0 = 10^{-2}$ and 10^{-3}, are plotted against the nondimensional shear rigidity factor g_D in Figs 8 and 9. The present solutions of equation (14) are also plotted in

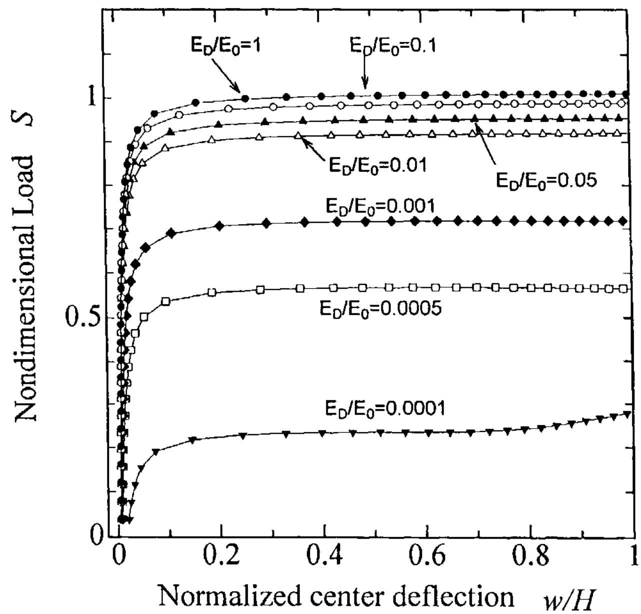

Figure 4. Relation between the nondimensional compressive load S and normalized center deflection w/h, when $E_0 = 100$ GPa, $2N = 4$, $l = 160$ mm, $h = 1$ mm, $t = 0.05$ mm.

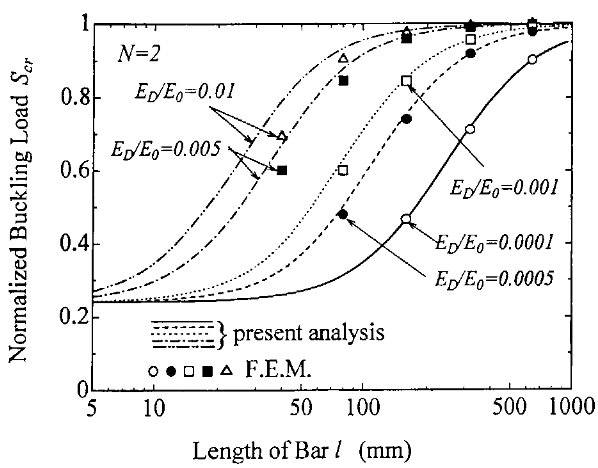

Figure 5. Nondimensional buckling load S_{cr} and length of the plate l, when $2N = 2$. The curves are the results from the present analysis and dotted marks are those from the finite element analysis.

Fig. 8 and agree well with the curve $g_D/(1 + g_D)$ even when $E_D/E_0 = 0.01$. The present solution is not plotted in Fig. 9 because the present solution approaches the curve $g_D/(1 + g_D)$ only with decrease of the relative modulus of the interleaves. The effect of bending stress in the interleaves can be neglected and the approximated value of equation (16) is sufficient to express the buckling load of the present

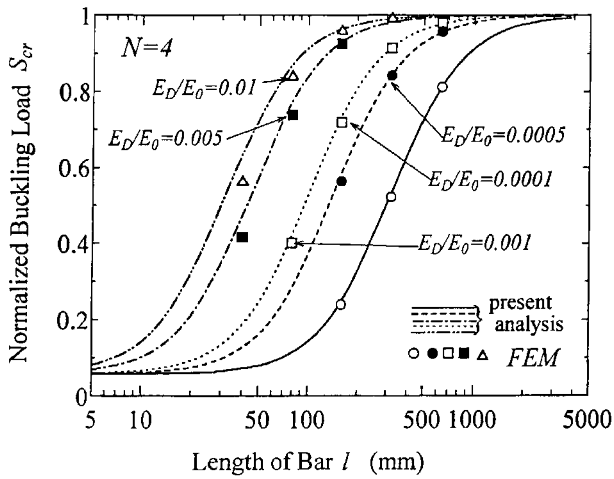

Figure 6. Nondimensional buckling load S_{cr} and length of the plate l, when $2N = 4$. The curves are the results from the present analysis and dotted marks are those from the finite element analysis.

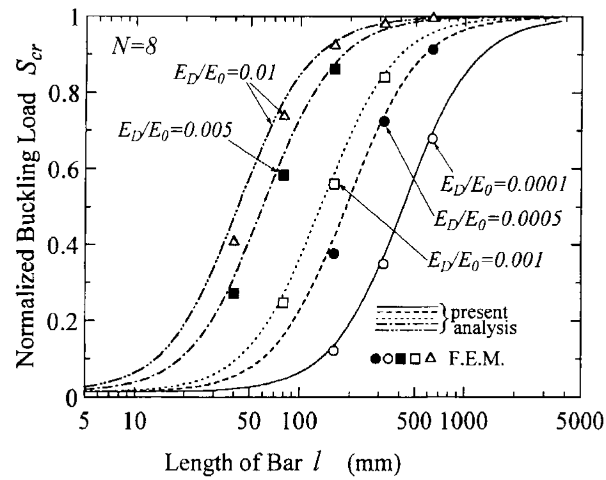

Figure 7. Nondimensional buckling load S_{cr} and length of the plate l, when $2N = 8$. The curves are the results from the present analysis and dotted marks are those from the finite element analysis.

problem. The buckling loads obtained from the finite element results are a little smaller than those obtained from equation (16). However, they still agree well with the finite element results. Equation (16) is sufficiently accurate to estimate the buckling load reduction due to the interleaves of very low modulus. The nondimensional shear rigidity factor of the interleaves g_D is dominant factor for the buckling load reduction. The assumption of the uniform shear deformation in a cross section may be said to give a rough estimate of reduced buckling load of the interleaved laminate.

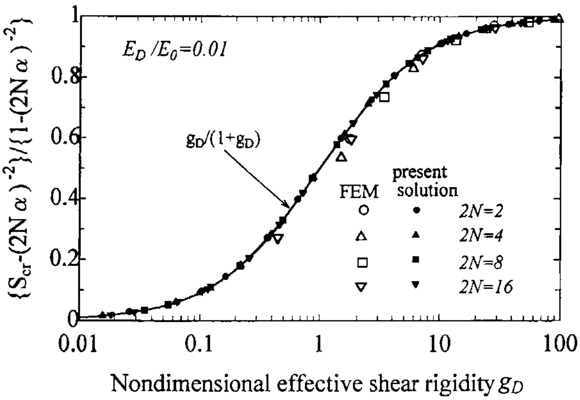

Figure 8. Relation between the nondimensional buckling load reduction and the nondimensional shear rigidity factor g_D, when $E_D/E_0 = 10^{-2}$.

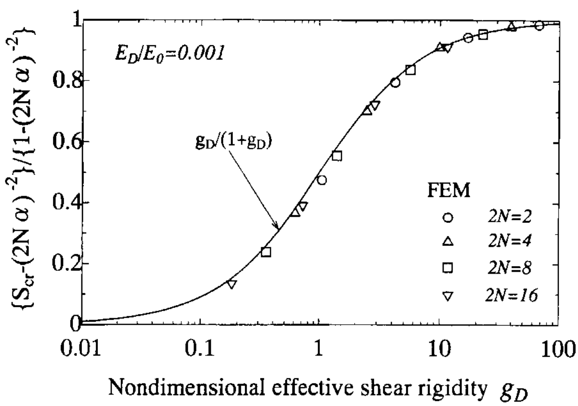

Figure 9. Relation between the nondimensional buckling load reduction and the nondimensional shear rigidity factor g_D, when $E_D/E_0 = 10^{-3}$.

5. CONCLUSION

We could derive a closed form equation for a rough estimate of reduced buckling loads of interleaved composite laminates with more than one soft interleave and found an important nondimensional shear rigidity factor by which the reduced buckling load of the plate with soft interleaves is strongly governed. The equation is accurate enough when the interleave is soft compared with the base layer material. The assumption of the uniform shear deformation in the interleaves is reasonable to obtain the buckling load of the laminates with soft interleaves. Buckling load significantly reduces by introducing many interleaves of very low modulus. The possible reduced buckling load can be as low as a sum of the buckling loads of the individual base layers separated by the soft interleaves.

The buckling load of the laminated beam of the other boundary conditions can be also estimated by using corresponding buckling mode functions. The buckling or compressive property reduction of more general cases of the interleaved laminates is supposed to be estimated based on the present simplified distribution of the inplane displacement u in the cross section.

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